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TURBULENT HEAT TRANSFER TO DILUTE POLYMER SOLUTIONS*

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Abstract—Heat-transfer rates are estimated for flows of dilute polymer solutions which exhibit drag reduction. The estimation, which is limited to the range of Prandtl numbers larger than one and smaller than fifty, is based on the semi-empirical analogy between heat and momentum transfer and on previous measurements of the velocity distribution in such fluids which show that drag reduction is accompanied by an increase of the laminar sublayer. The analysis indicates that the Nusselt number of the flow decreases whenever the friction coefficient does. A relation between the Nusselt number of such flows, the Reynolds number, the Prandtl and the friction coefficient is established and compared with experimental results.

NOMENCLATURE

C,	a concentration dependent parameter, see equation (15);
c_p,	specific heat;
D,	diameter of pipe;
Db,	Deborah number, $Db = t_1 u_b^2/\nu$;
f,	friction coefficient as defined by Darcy–Weisbach, $f = 8\tau_w/\rho u_b^2$;
k,	thermal conductivity;
k',	von Kármán's constant;
Nu,	Nusselt number, see equations (34) and (35);
Pc,	concentration of polymers in ppm;
PN,	polymer number, $PN = \sqrt{(t_1 \nu)}/2R$;
Pr,	Prandtl number;
q,	average heat flux;
q_w,	average heat flux at the wall;
R,	radius of pipe;
R^+,	dimensionless radius, $R^+ = R\sqrt{(\tau_w/\rho)}/\rho$;
Re,	Reynolds number;
Re_r,	threshold Reynolds number;
St,	Stanton number;
T,	temperature;
T_w,	temperature at the wall;
T^+,	dimensionless temperature defined in equation (28);
T_b^+,	dimensionless bulk temperature defined in equation (33);
t_1,	characteristic time of polymer, see note after equation (14);
u,	velocity;
u_b,	average velocity, or bulk velocity;
u^+,	dimensionless velocity, $u/\sqrt{(\tau_w/\rho)}$;
Δu^+,	upward shift of the logarithmic velocity profile, see Fig. 3;
y,	distance from the wall;
y^+,	dimensionless distance from the wall, $y^+ = y\sqrt{(\tau_w/\rho)}/\nu$;
y_p,	the distance from the wall where equations (6) and (7) intersect;
y_1,	edge of laminar sublayer;
y_2,	edge of buffer zone.

Greek symbols

ϵ, ϵ_h,	eddy viscosity and diffusivity (assumed to be equal);
μ,	viscosity;
ν,	kinematic viscosity;
ν_0,	kinematic viscosity of the pure solvent;
ρ,	specific density;
τ,	shear stress (including the Reynolds shear stress);
τ_w,	shear stress at the wall.

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* The work is based in part on a M.Sc. thesis of the second author.

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INTRODUCTION

THE DISCOVERY that the addition of a minute quantity of certain polymers to turbulent fluid flow causes a reduction of the friction coefficient has caused interest in the potential application of such additives in various branches of engineering. An important application of fluid flow is in heat-transfer systems and it is therefore worthwhile to examine the effect of the polymer additives on heat-transfer characteristics of turbulent flows. Such an investigation would also contribute to the understanding of the phenomenon of the drag reduction.

Several investigations of the subject have been reported recently and semi-empirical expressions for heat-transfer coefficients have been proposed.

In 1958 Metzner and Friend [1] have modified Reichardt's analogy of heat transfer and proposed a semi-empirical relation which provided a very good agreement with experimental data for Newtonian and purely viscous non-Newtonian fluids:

$$St = \frac{f/2}{1\cdot 2 + 11\cdot 8\,(Pr - 1)(Pr)^{-\frac{1}{3}}\sqrt{(f/2)}}. \quad (1)$$

It was hoped that this expression would be applicable in elastico-viscous fluids as well, however, Pruitt, Whitsitt and Crawford [2] have already shown that it does not, as demonstrated in Fig. 1. Pruitt *et al.* have empirically found that a good correlation with data for viscoelastic fluids is achieved by modifying equation (1) as follows:

$$St = \frac{f(1 - FR)/2}{1\cdot 2 + 11\cdot 8\,(Pr - 1)\,Pr^{-\frac{1}{3}}\sqrt{[f(1 - FR)/2]}} \quad (2)$$

where the parameter *FR* is defined by the equation

$$FR = [(\Delta P_{\text{base}} - \Delta P_{\text{base+polymer}})/\Delta P_{\text{base}}]_v \quad (3)$$

in which ΔP is the pressure drop along the test section for the base fluid and for the polymer solution, at the same bulk velocity v. The correlation between equation (2) and the experimental data is demonstrated in Fig. 2. An alternative modification was also proposed by the same authors, according to which *FR* was measured at the point of maximum drag reduction.

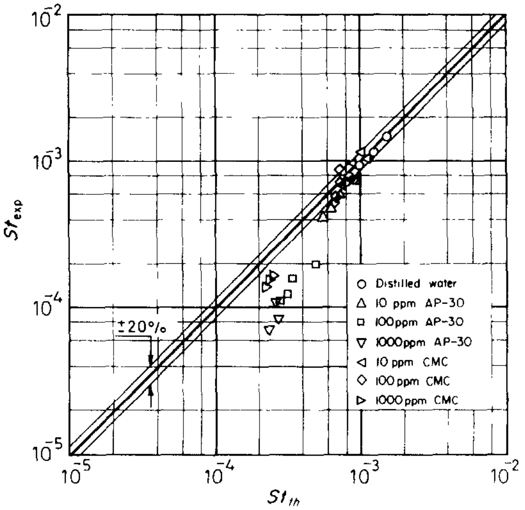

FIG. 1. Comparison of calculated Stanton numbers, using equation (1) with measurements. (From Pruitt *et al.* [2]).

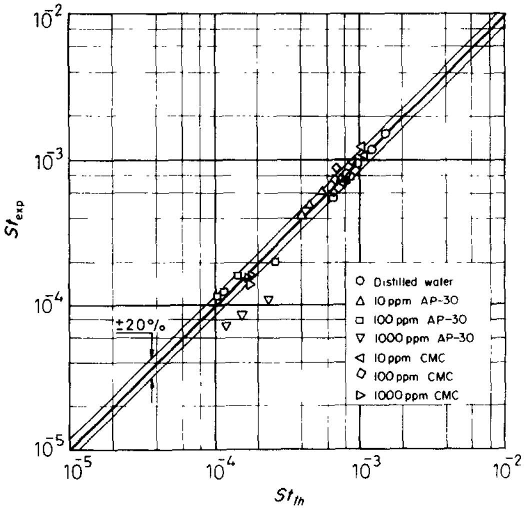

FIG. 2. Comparison of calculating Stanton number using equation (2) with measurements (from Pruitt *et al.* [2]).

The analysis of heat transfer in dilute polymer solutions presented in this paper is similar to that of von Kármán [3]. The flow is divided into three regions, a laminar sublayer, a buffer zone and a turbulent core. The limits of the three regions were determined by generalizing von Kármán's model so that the velocity distribution in the turbulent core would fit the measurements of Elata, Lehrer and Kahanovitz [4] in pipe flow of dilute polymer solutions. It is obvious that such a model would fail in case of high Prandtl number fluids, as did the original model of von Kármán. In order to predict heat transfer in this range, a more complicated velocity or eddy viscosity distribution which should take into consideration the turbulence in the sublayer, should be used. Since there are no measurements of the velocity distribution near the wall and the effect of the polymers in this region is not known, and since there are no data of heat transfer in polymer solutions at very high Prandtl numbers, it was felt that it would be justified to adopt a simple model in order to examine the gross features of the phenomenon.

We shall see that the model adopted predicts, within a reasonable error, the experimental data, in which the Prandtl number varies between three and fifty.

In the process of reviewing the paper, another analysis of heat transfer in drag reducing fluids by Wells [9] was brought to the attention of the authors. The results of Wells' analysis are similar to those presented in this paper.

VELOCITY DISTRIBUTION AND FRICTION COEFFICIENT

Measurements of velocity distribution in pipe flow of Guar-gum dissolved in water for a large range of concentrations and different pipe sizes were reported and analyzed by Elata *et al.* [4] and by Meyer [5]. Typical measurements from [4] are described in dimensionless form in Fig. 3 where

$$y^+ = y\sqrt{(\tau_w/\rho)}/\nu \quad \text{and} \quad u^+ = u/\sqrt{(\tau_w/\rho)}. \quad (4)$$

The measurements indicate that the flow of dilute polymer solutions in the fully turbulent region of the pipe is similar to that of regular fluids without additives, in the sense that the velocity profile is described by the same velocity defect-law

$$u^+ - u_j^+ = 1/k' \ln (y^+/y_j^+) \qquad (5)$$

where k' is a universal constant, approximately equal to 0·4. The difference between the flow of polymer solutions and the flow of regular fluids is in the value of u_j^+ and y_j^+. This difference is better demonstrated when one expresses equation (5) in the form of the law-of-the-wall, which gives:

$$u^+ = 2{\cdot}5 \ln y^+ + 5{\cdot}5 + \Delta u^+. \qquad (6)$$

The term Δu^+ denotes the upward shift of the logarithmic profile with respect to the universal logarithmic profile for smooth pipe flow of regular fluids which is described by equation (6) with $\Delta u^+ = 0$. Now, the laminar flow of polymer solutions is identical to laminar flow of Newtonian fluids and one may assume that there exists a "laminar" sublayer close to the wall, where

$$u^+ = y^+. \qquad (7)$$

The zone between the laminar sublayer and the fully turbulent region is called the buffer zone. Measurements in the buffer zone and in the laminar sublayer in case of polymer solutions, are not available, but the friction coefficient can

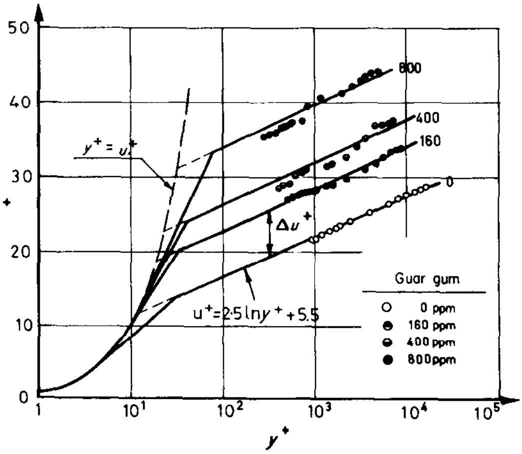

FIG. 3. Typical velocity profiles of Guar-gum solutions.

be estimated closely by assuming that the flow is divided into two regions: a laminar sublayer in the region $0 < y^+ < y_j^+$ where equation (7) is valid, and a turbulent region $y_j^+ < y^+ < R^+$ where equation (5) is valid. The dimensionless distance from the wall y_j^+ is related to Δu^+ by the equation:

$$y_j^+ - 2.5 \ln y_j^+ = \Delta u^+ + 5.5. \tag{8}$$

The friction coefficient f, as defined by Darcy–Weissbach, is given by the equation:

$$\frac{\tau_w}{\rho} = \frac{f}{8} u_b^2 \tag{9}$$

where $u_b = Q/\pi R^2$ is the average velocity or bulk velocity. (It should be noted that the friction coefficient defined by Darcy–Weissbach is four times larger than that defined by Fanning.) Expressing u_b in terms of the dimensionless variables defined earlier one finds that

$$\sqrt{(8/f)} = u_b^+ = \frac{2}{R^{+2}} \int_0^{R+} u^+(R^+ - y^+)\,\mathrm{d}y^+. \tag{10}$$

Since the discharge in the laminar and buffer zone is relatively small one can evaluate f from equation (10) assuming that equation (5) is valid throughout the cross-section. In this manner one finds that:

$$\sqrt{(8/f)} = 2.5 \ln R^+ + 1.75 + \Delta u^+ \tag{11}$$

or

$$\sqrt{(8/f)} = 2.5 \ln (R^+/y_j^+) + y_j^+ - 3.75. \tag{12}$$

Equation (11) may also be expressed in this form:

$$\sqrt{(8/f)} = 2.5 \ln [Re \sqrt{(f/32)}] + 1.75 + \Delta u^+ \tag{13}$$

where $Re = 2u_b R/\nu$ is the Reynolds number of the flow.

Elata *et al.* [4] have found in their experiments that the shift Δu^+ is related to the characteristics of the flow at the wall as follows:

$$\Delta u^+ = 2.5C \ln (\tau_w t_1/\mu) \tag{14}$$

where t_1 is a characteristic relaxation time of the polymer molecules, μ the viscosity of the solution, and C a concentration dependent parameter. The reader should note that the characteristic time t_1 in this model is assumed to be a constant for a given polymer. This assumption has not yet been proved to be applicable over a wide range of viscosities.

According to the measurements [4] in Guar-gum, C increases linearly with concentration for very dilute solutions as described by the equation

$$C = 0.0226\, Pc, \tag{15}$$

where Pc is the concentration in ppm. Deviation from equation (15) starts at around 1000 ppm and a maximum value was found at 3000 ppm.

The shift Δu^+ may also be expressed in the following form:

$$\Delta u^+ = 2.5C \ln (f Db/8) \tag{16}$$

and

$$\Delta u^+ = 2.5C \ln (Re^2 f PN^2/8) \tag{17}$$

where $Db = t_1 u_b^2/\nu$ is the Deborah number and $PN = (t_1 \nu)^{\frac{1}{2}}/D$ is a dimensionless number which we shall term the polymer number; $(PN^2 = Db/Re^2)$.

The friction coefficient may thus be expressed in terms of the polymer properties as follows:

$$\sqrt{(8/f)} = 2.5 \ln [0.356 (Re \sqrt{f})^{1+2C} \times (PN^2/8)^C]. \tag{18}$$

We see from equations (14) and (13) that drag reduction occurs only when

$$t_1 \tau_w/\mu > 1, \tag{19}$$

namely, when the characteristic relaxation time of the polymer molecule is larger than the time scale of the boundary layer, μ/τ_w.

The threshold Reynolds number Re_r, which denotes the beginning of drag reduction, and the threshold Deborah number can be found by using equations (16) and (13) with $\Delta u^+ = 0$ which indicate that

$$Re_r = F_1(PN) \quad \text{and} \quad Db_r = F_2(PN). \tag{20}$$

The functions F_1 and F_2 are described in Fig. 4. It is obvious from this figure that PN would be a convenient parameter whenever the threshold velocity, for a given solution and pipe diameter, has to be calculated.

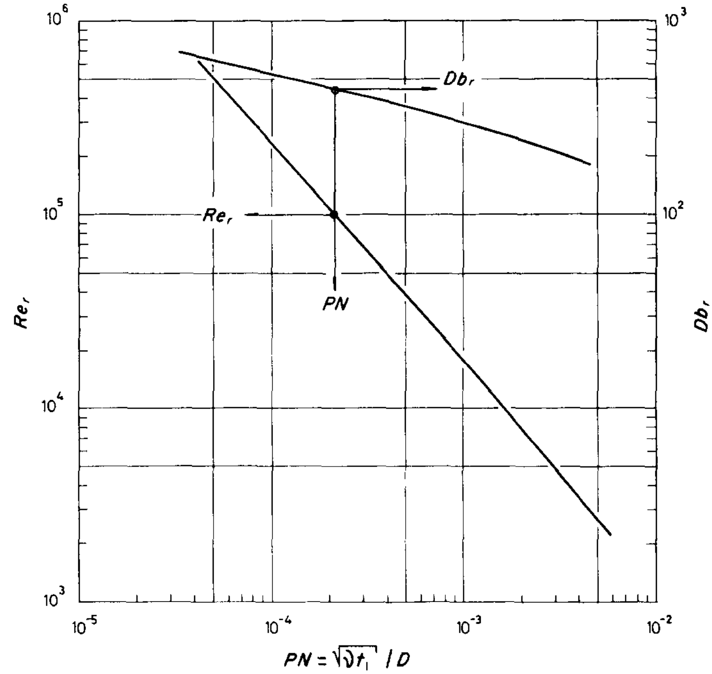

FIG. 4. Dependence of threshold Reynolds and Deborah numbers on the polymer number.

ANALYSIS OF HEAT TRANSFER

Time averaging the equations of motion and heat by the conventional method of Reynolds, one finds that the apparent average shear stress τ and average heat flux q in a boundary layer are given by:

$$\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y} - \rho\overline{u'v'} \tag{21}$$

and

$$q = -k\frac{\mathrm{d}T}{\mathrm{d}y} + \rho c_p \overline{T'v'}. \tag{22}$$

The term $-\rho\overline{u'v'}$ represents the apparent turbulent shear stress, known as the Reynolds stress. The term $\rho c_p\overline{T'v'}$ represents the turbulent heat flux. Introducing an eddy viscosity ϵ and an eddy diffusivity ϵ_h for turbulent momentum and heat transfer, equations (21) and (22) may be rewritten as follows:

$$\tau = \rho(\nu + \epsilon)\frac{\mathrm{d}u}{\mathrm{d}y} \tag{23}$$

$$q = -(k + \rho c_p \epsilon_h)\frac{\mathrm{d}T}{\mathrm{d}y}. \tag{24}$$

The ratio ϵ/ϵ_h is referred to as the turbulent Prandtl number, Pr_t. Following Reynolds, it is assumed that the turbulent transfer of heat and turbulent transfer of momentum are similar so that

$$Pr_t = \frac{\epsilon}{\epsilon_h} = 1. \tag{25}$$

If the properties of the fluid are assumed to be constant, equations (23) and (24) may be expressed as:

$$\frac{\tau}{\tau_w} = \left(1 + \frac{\epsilon}{\nu}\right)\frac{du^+}{dy^+} \tag{26}$$

and

$$\frac{q}{q_w} = \left(\frac{1}{Pr} + \frac{\epsilon}{\nu}\right)\frac{dT^+}{dy^+} \tag{27}$$

where

$$T^+ = (T_w - T)\,c_p\tau_w/[q_w\sqrt{(\tau_w/\rho)}]. \tag{28}$$

Since the variation of the heat flux and shear with distance from the wall has only a slight effect on the temperature or velocity distribution, we shall adopt the assumption, originally proposed by Prandtl and successfully applied in predicting heat-transfer coefficients by many other investigators, that

$$\frac{\tau}{\tau_w} = 1 \quad \text{and} \quad \frac{q}{q_w} = 1. \tag{29}$$

It follows that

$$\left(1 + \frac{\epsilon}{\nu}\right)\frac{du^+}{dy^+} = \left(\frac{1}{Pr} + \frac{\epsilon}{\nu}\right)\frac{dT^+}{dy^+}. \tag{30}$$

Equation (30) permits us to predict the temperature distribution whenever the velocity distribution is known.

In the region away from the wall, where the molecular shear stress and heat transfer are relatively small and may be neglected, one finds that

$$T^+ - T_j^+ = u^+ = u_j^+, \tag{31}$$

where T_j^+ and u_j^+ are the dimensionless temperature and velocity at some distance from the wall $y^+ = y_j^+$.

In the region close to the wall, where the turbulent transfer of momentum and heat may be neglected, integration of equation (30) gives:

$$T^+ = Pr\,u^+. \tag{32}$$

In the buffer zone both molecular and transport processes are important and the temperature distribution can be obtained only when the distribution of u^+ is specified.

The dimensionless bulk temperature T_b^+ is defined as

$$T_b^+ = \frac{\int_0^{R^+} T^+u^+(R^+ - y^+)\,dy^+}{\int_0^{R^+} u^+(R^+ - y^+)\,dy^+} \tag{33}$$

and the Nusselt number is defined as

$$Nu = \frac{2Rq_w}{(T_w - T_b)k}. \tag{34}$$

It follows from the previous definitions that

$$Nu = \frac{2Pr\,R^+}{T_b^+}. \tag{35}$$

In order to calculate the Nusselt number it is necessary to find T_b^+ by integrating the temperature distribution over the cross-section of the tube using equation (33). This procedure is analogous to the one of determining the friction coefficient by integrating equation (10). In this case too, one could estimate T_b quite closely by assuming that the logarithmic laws, equations (5) and (31), are valid throughout the cross-section, but it is necessary, at first, to determine the value of T_j^+.

Following the analysis of Prandtl and Taylor [8], the flow may be divided into two regions. In the laminar region

$$u^+ = y^+ \tag{36}$$

and thus

$$T^+ = Pr\,y^+. \tag{37}$$

Substituting $T_j^+ = Pr\,y_j^+$ in equation (31), one finds that the temperature within the turbulent core is given by

$$T^+ = u^+ + y_j^+(Pr - 1). \tag{38}$$

Previous works on heat transfer in Newtonian fluids have already shown that the above procedure is valid only when the Prandtl number is close to one. Whenever the Prandtl number is higher, one may not neglect the existence of the buffer zone. The higher the Prandtl number the more sensitive integration of (30) is to the form of assumed velocity distribution in the buffer zone.

Diluted polymer solutions have a Prandtl number larger than one. It is therefore necessary to make some assumptions on the nature of the velocity distribution in the buffer zone. Since measurements in this region are not available, any assumption would be speculative. Moreover, previous analyses of heat transfer in Newtonian liquids show that substantial differences in the value of Nusselt number result from different expressions for the velocity distribution, all of which seem to fit the available experimental data quite well. We shall therefore prefer at this stage to describe the buffer zone by a simple model. The proposed model, which is illustrated in Fig. 5, is a modification of the one used by von Kármán [3]. Von Kármán divided the flow into the following three regions:

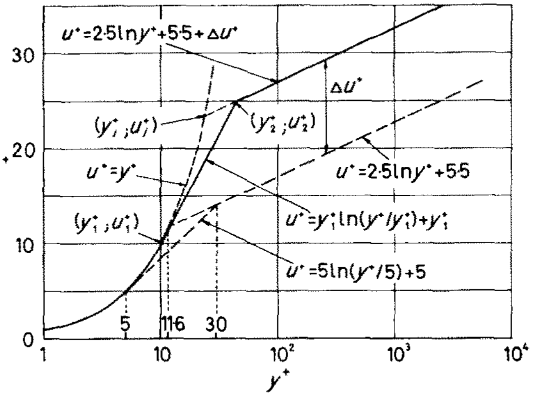

FIG. 5. Illustration of assumed velocity profile.

a laminar sublayer:

$$0 < y^+ < 5 \qquad \text{where} \quad u^+ = y^+ \tag{39}$$

a buffer zone:

$$5 < y^+ < 30 \qquad \text{where} \quad u^+ = 5 \ln y^+ - 3{\cdot}05 \tag{40}$$

a turbulent zone:

$$30 < y^+ < R^+ \qquad \text{where} \quad u^+ = 2{\cdot}5 \ln y^+ + 5{\cdot}5. \tag{41}$$

The velocity profile is continuous throughout the layer, however, the velocity derivative is not continuous at the edge of the buffer zone. Denoting the edge of the laminar sublayer by y_1^+, and the edge of the buffer zone by y_2^+, the velocity distribution in the buffer zone, $y_1^+ < y^+ < y_2^+$, may be expressed as:

$$u^+ = y_1^+ \ln(y^+/y_1^+) + y_1^+. \tag{42}$$

The intersection of equation (40) with the curve $u^+ = y^+$ is denoted by y_j^+. According to von Kármán $y_j^+ = 11{\cdot}6$, and for Newtonian fluids the ratio of y_1^+/y_j^+ is therefore

$$y_1^+/y_j^+ = 0{\cdot}43. \tag{43}$$

We shall assume now, that in the case of polymer solutions the ratio y_1^+/y_j^+ remains 0·43, and further assume that the velocity distribution in the buffer zone is described by equation (42). Now, the velocity distribution in the turbulent region $y_2^+ < y^+ < R^+$, is described by the logarithmic profile [equation (5)],

$$u^+ = 2{\cdot}5 \ln(y^+/y_j^+) + u_j^+, \tag{44}$$

and the value of y_2^+ may therefore be determined by the intersection of equations (42) and (44). It follows that the thickness of the buffer zone is given by the expression

$$\ln(y_2^+/y_1^+) = \frac{1{\cdot}32 y_j^+ - 4{\cdot}9}{y_j^+ - 5{\cdot}8}; \qquad (y_j > 11{\cdot}6). \tag{45}$$

We see that the velocity distribution in our model depends on one parameter, y_j^+, which is related to Δu^+ and the polymer properties by equations (8) and (14). We have already seen that the dimensionless temperature at y_1^+, is given by

$$T_1^+ = Pr\, y_1^+. \tag{46}$$

The dimensionless temperature at the edge of the buffer zone is found from equation (30), using equations (42) and (29) to evaluate ϵ/ν and du^+/dy^+:

$$T_2^+ = y_1^+\{Pr + \ln[Pr(y_2^+/y_1^+ - 1) + 1]\}. \quad (47)$$

The dimensionless velocity u_2^+ at this point is given by

$$u_2^+ = y_1^+ \ln y_2^+/y_1^+ + y_1^+. \quad (48)$$

Now that T_2^+ and u_2^+ are known, we may express the temperature distribution in the turbulent region by the equation

$$T^+ = T_2^+ + u^+ - u_2^+. \quad (49)$$

The dimensionless bulk temperature will now be evaluated using equation (33). Since the contribution of the temperature in the laminar sublayer and the buffer zone is relatively small we shall calculate T_b^+ by assuming that equation (49) is valid throughout the cross-section. It is found, in this manner, that

$$\begin{aligned} T_b^+ &= y_1^+\{\ln[Pr\, y_2^+/y_1^+ - (Pr - 1)] \\ &\quad + (Pr - 1) - \ln y_2^+/y_1^+\} \\ &\quad + \int_0^{R^+} (2{\cdot}5 \ln y^+/y_j^+ + y_j^+)^2 (R^+ - y^+)\, dy^+ / \\ &\quad \int_0^{R^+} (2{\cdot}5 \ln y^+/y_j^+ + y_j^+)(R^+ - y^+)\, dy^+. \end{aligned} \quad (50)$$

Integrating (50) and substituting in equation (35), using the relation $R^+ = Re\sqrt{(f/32)}$, the following expression for the Nusselt number is obtained:

$$\begin{aligned} Nu = Pr\, Re \sqrt{(f/8)}/\{y_1^+(\ln[Pr - (Pr - 1) \\ \times y_1^+/y_2^+] + (Pr - 1)) + \sqrt{(8/f)} \\ + 125[(f/8)]/16\}. \end{aligned} \quad (51)$$

Since f is usually smaller than 0·03, the last term in the denominator of equation (51) may be neglected. Rearranging terms, one finds that the Stanton number is given by the equation:

$$\begin{aligned} St = Nu/Pr\, Re = \sqrt{(f/8)}/\{y_1^+(\ln[Pr \\ - (Pr - 1)\, y_1^+/y_2^+] + (Pr - 1)) + \sqrt{(8/f)}\}. \end{aligned} \quad (52)$$

THEORETICAL RESULTS

We shall examine now the effect, according to our model, of the various parameters on the friction and heat transfer.

The effect of the polymer concentration on the friction and Nusselt number is demonstrated in Fig. 6. In calculating this figure it has been assumed that the Prandtl number, pipe diameter, polymer properties and the solvent's viscosity are constants. The polymer number $PN = (\nu t)^{\frac{1}{2}}/D$ is not constant, however, and so is the threshold Reynolds number, due to the dependence of the viscosity of the solution and the parameter C on the concentration, which were taken from experiments in Guar-gum.

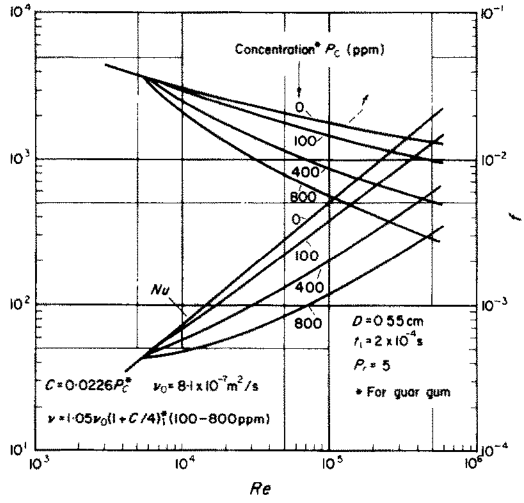

FIG. 6. Typical effect of concentration on friction coefficient and Nusselt number.

The effect of the Prandtl number on the Nusselt number of the flow is shown in Fig. 7. We see that the Prandtl number affects the heat transfer to the flow of dilute polymer solutions exactly as it affects the heat transfer to the solvent.

The dependence of the friction and heat transfer on the polymer number PN is demonstrated in Fig. 8. The other properties were taken to be constants. This figure clearly shows that the polymer is effective only beyond the threshold Reynolds number which can be decreased by increasing the polymer number.

A comparison between the relative friction reduction and the relative Nusselt, or Stanton number reduction, *at the same Reynolds and Prandtl numbers*, is presented in Fig. 9 where $St_{\text{solution}}/St_{\text{solvent}}$ which equals $Nu_{\text{solution}}/Nu_{\text{solvent}}$ is plotted vs. $f_{\text{solution}}/f_{\text{solvent}}$.

We see that in the special case $Pr = 1$, the relative friction reduction is equal to the relative heat-transfer reduction. In fact, it is easy to see from equation (52) that for $Pr = 1$,

$$St = f/8. \tag{53}$$

For higher Prandtl numbers, the reduction of the heat-transfer coefficients is larger than the reduction of the friction coefficients. Also when $Pr > 1$ the relation between these ratios is slightly dependent on the Reynolds number. It should be noted that these ratios do not describe the actual changes which take place in a particular system when polymers are added to the flow, since in these curves the Reynolds and Prandtl numbers of the solution and solvent are assumed to be the same. On the other hand, these relations are independent of the polymer properties, concentration, or polymer number.

We have shown in equation (52) that the Stanton or Nusselt numbers are functions of Pr, Re, y_1^+, y_2^+ and f. The parameters y_1^+ and y_2^+ are determined by Δu^+ which according to equation (13) is related to f and Re. We may therefore conclude that a definite relation exists

$$Nu = Nu(Pr, Re, f), \tag{54}$$

independent of the polymer properties.

Now, in obtaining the relation (54) we need not use equations (14) and (15), which relate the friction coefficient to the polymer properties. That means, that equation (54) should be valid even if the drag reduction is related to different polymer properties provided, of course, that our assumptions on the velocity distribution are correct. This is a very important conclusion in view of the fact that different theories have been proposed to explain the mechanism of drag reduction. Unfortunately, equation (54) cannot be expressed in an explicit form and

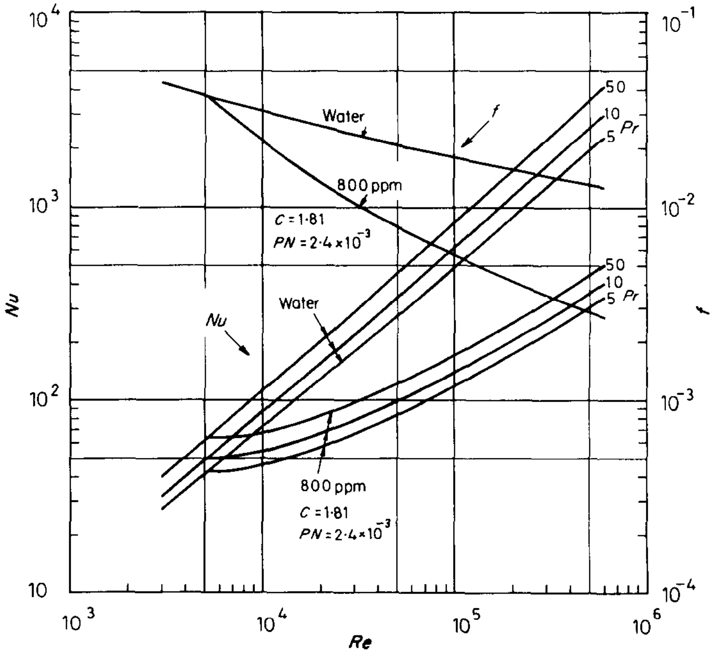

FIG. 7. Effect of the Prandtl number on the heat transfer.

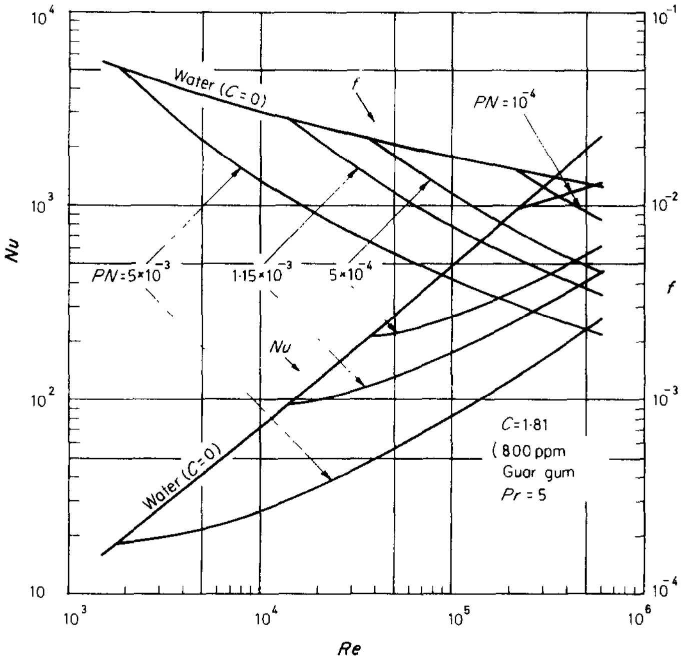

FIG. 8. Friction coefficient and Nusselt number curves at different polymer numbers.

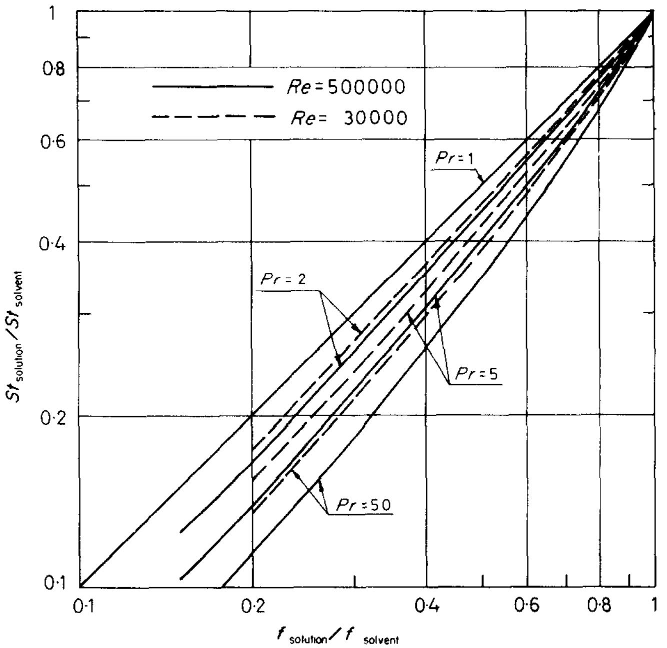

FIG. 9. Relation between relative reduction of Stanton number and friction coefficient at the same Prandtl and Reynolds numbers.

since it includes four variables it can neither be described graphically.

In Fig. 10 we have described the relation between St, Pr and f at Reynolds numbers of 30000 and 500000. Obviously the highest value of f in each case is that of the solvent.

Figure 11 demonstrates the dependence of the Stanton number on the Reynolds number of the flow at different Prandtl numbers, for a given friction coefficient $f = 0{\cdot}01$. Similar results were obtained for different values of f, which shows that the variation of St with Re for a given friction coefficient is quite small. In equation (2), which was proposed by Pruitt *et al.*, the Reynolds number did not appear at all, however, another parameter, Pr, was included.

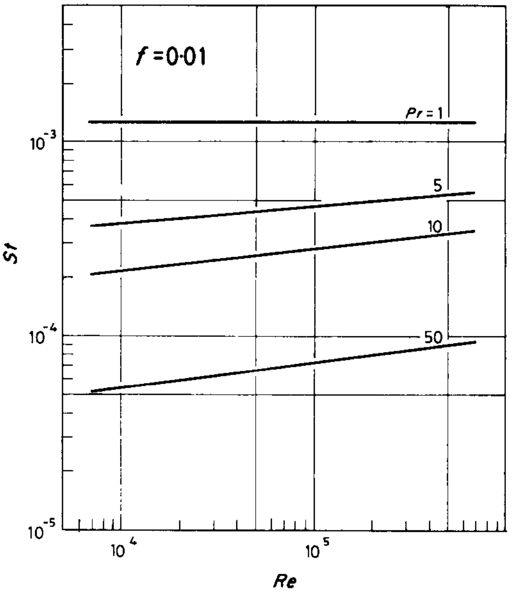

FIG. 11. Stanton number as a function of the Reynolds and Prandtl numbers ($f = 0{\cdot}01$).

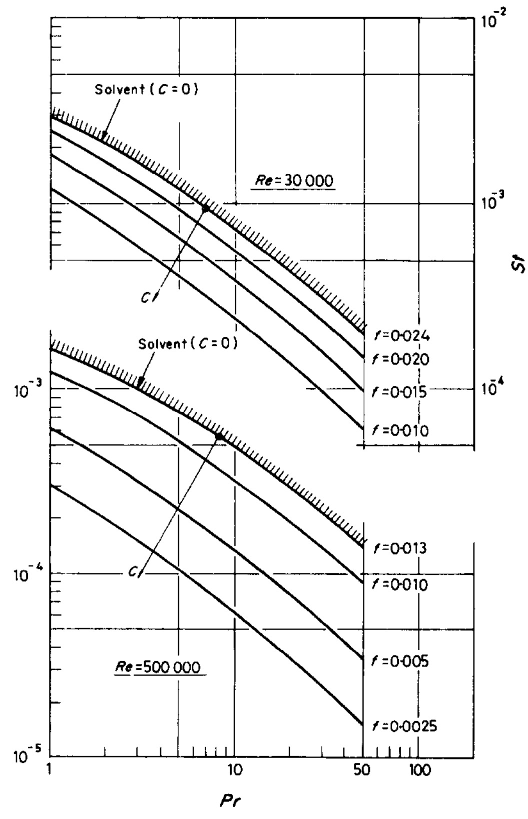

FIG. 10. Stanton number as a function of the friction coefficient and Prandtl number (Re = 30000 and 500000).

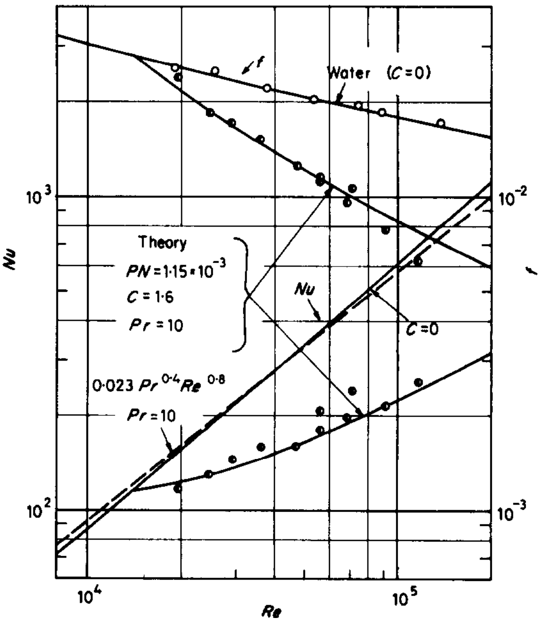

FIG. 12. Measurements of Nusselt number and friction coefficient in a 500 ppm ET-597 solution ($Pr \cong 11$).

COMPARISON WITH EXPERIMENTAL RESULTS

The results of the semi-empirical analysis will be compared with experimental data reported in three investigations of fully developed turbulent heat transfer in pipe flow:

(1) A thesis by Gupta [6] which reports measurements with ET 597, a high molecular weight, partially hydrolyzed polymer dissolved in water. Three concentrations were used, 100, 500 and 4500 ppm, but we shall not refer to the experiments with 4500 ppm solution as the flow was mostly laminar.

(2) An investigation by Pruitt *et al.* [2], in which carboxymethy cellulose (CMC-7H), and separan (AP-30) solutions in water at concentrations of 10, 100 and 1000 ppm were used.

(3) Measurements by the authors which are described in the thesis of Paz [7], using Guar-gum solution in water at concentrations of 100, 400 and 800 ppm.

Measurements with pure water, which were reported in these investigations were also included in the comparison with the analytical results.

The authors have found it difficult to compare the experimental results with the theory by plotting the experimental data in the figures which describe the theoretical results. The main reason is that *all* the independent parameters vary from one experimental point to another. When the Reynolds number of the flows was varied, the wall temperature, the bulk temperature and the viscosity varied with it, which changed the Prandtl number and polymer number. Moreover, the value t_1 for the various polymers and its dependence on the temperature are not known. The experiments with 500 ppm ET 597 [6], however, were found to be within a small range of Prandtl numbers (10·8–11·6), and one could therefore compare the variation of the Nusselt number with the Reynolds number in this case with the theoretical curve. Figure 12 shows that the measurements of the friction and the Nusselt number are fairly well described by the theoretical curves with $PN = 1{\cdot}15 \times 10^{-3}$, and $C = 1{\cdot}6$. The effective kinematic viscosity was estimated to be $1{\cdot}7 \times 10^{-6}$ m^2/s which suggests a characteristic time $t_1 = 2{\cdot}68 \times 10^{-4}$ s. We did not plot in this figure the data of heat transfer in pure water since the Prandtl number in this case varies between 4 and 6·5, however, we have included in the same curve the semi-empirical relation $Nu = 0{\cdot}023\ Re^{0{\cdot}8}\ Pr^{0{\cdot}4}$ for comparison with the curve for $C = 0$.

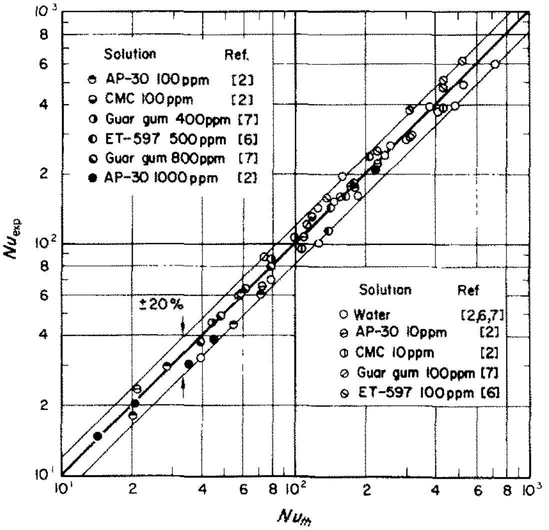

FIG. 13. Comparison of calculated Stanton number using authors' model with measurements.

In Fig. 13 we have plotted the theoretical values of the Nusselt number, calculated from the measured values of the friction coefficients, Prandtl and Reynolds numbers using our theory, vs. the experimental values. Part of the data is for the water and for dilute polymer flowing in the range where drag reduction does not occur and which appear in the studies [2, 6, 7]. The correlation between the theoretical and the experimental values, except for the measurements with 1000 ppm CMC which were not included, is reasonable and almost all the experimental data are predicted by the theory within an error of ±20 per cent. The deviation of the data from the empirical relation of

Pruitt *et al.*, see Fig. 2, is of the same order, except for the measurements with 1000 ppm AP-30. Such a correlation should be regarded as satisfactory due to several reasons: we have assumed constant fluid properties. In reality the temperature changes with the distance from the wall and the fluid properties are not constant. Now, the temperature differences in the three experimental studies were not very high, but since the fluid properties which appear in the dimensionless numbers were determined in each study in a slightly different manner, additional scatter has probably resulted.

The viscosity of the solution is slightly shear dependant, and it was therefore estimated from the rheograms according to the value of the shear at the wall. Scatter, and possible error, might have also been caused by the assumption used in the three investigations, that the thermal conductivity of the solution is equal to that of the solvent. The validity of this assumption has already been questioned by Pruitt *et al.* [2].

One should recall, however, that most of the data are in the range of $3 < Pr < 50$ and that the above analysis is limited to this range of Prandtl numbers.

CONCLUSIONS

Friction and heat-transfer coefficients in turbulent flows of dilute polymer solutions which exhibit drag reduction can be correlated using the semi-empirical analogy between momentum and heat transfer.

The analysis shows that frictional drag reduction is always accompanied by a reduction of the heat-transfer coefficient. The relative reduction of the Nusselt number, for given Reynolds and Prandtl numbers ($Pr > 1$), was found to be larger than the relative reduction of the friction coefficient, particularly at high Reynolds numbers.

The assumptions concerning the velocity distribution near the wall were found to be satisfactory for predicting heat-transfer rates, but there is no doubt that the present theory should be refined as soon as measurements near the wall are available.

The correlation between the theory and the experimental data of heat-transfer also suggests that the logarithmic velocity profiles used by Elata *et al.* [4] and Meyer [5] are universal for flows of dilute polymer solutions which exhibit drag reduction.

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Résumé—Les vitesses de transport de chaleur sont estimées pour des écoulements de solutions diluées de polymères qui présentent une diminution de traînée. L'estimation, qui est limitée à la gamme des nombres de Prandtl supérieurs à l'unité et plus petits que 50, et basée sur l'analogie semi-empirique entre les transports de chaleur et de quantité de mouvement et sur des mesures antérieures de la distribution de vitesse dans de tels fluides qui montrent que la diminution de traînée est accompagnée par une augmentation de la sous-couche laminaire. L'analyse indique que le nombre de Nusselt de l'écoulement diminue toutes les fois que le coefficient de frottement le fait. On a établi une relation entre le nombre de Nusselt de tels écoulements, les nombres de Reynolds et de Prandtl et le coefficient de frottement et on l'a comparée avec des résultats expérimentaux.

Zusammenfassung—Der Wärmeübergang wurde bestimmt bei der Strömung von verdünnten Polymerlösungen, die eine Verminderung des Schleppwiderstandes aufweisen. Diese Ermittlung, die auf den Bereich von Prandtl-Zahlen zwischen 1–50 beschränkt war, beruhte auf einer halbempirischen Analogie zwischen Wärme- und Impulstransport und auf früheren Messungen der Geschwindigkeitsverteilung in solchen Flüssigkeiten, welche eine Erniedrigung des Schleppwiderstandes zusammen mit einer Vergrösserung der laminaren Unterschicht aufweisen. Die Analyse zeigte, dass die Nusselt-Zahl der Strömung abnimmt mit dem Reibungskoeffizienten. Eine Beziehung zwischen Nusselt-Zahl, Reynolds-Zahl, Prandtl-Zahl und Reibungskoeffizient wurde aufgestellt und mit Versuchsergebnissen verglichen.

Аннотация—Определены тепловые потоки в случае течения разбавленных растворов полимеров, для которых характерно пониженное сопротивление. Оценка, ограниченная областью изменения чисел Прандтля $Pr > 1$ и $Pr < 50$, основсываетя на полуэмпирической аналогии между переносом тепла и количества движения и на предыдущих измерениях распределения скорости в таких жидкостях, которые показывают, что уменьшение сопротивления сопровождается увеличением ламинарного подслоя. Из анализа следует, что число Нуссельта для потока уменьшается с уменьшением коэффициента трения. Установлена зависимость между Nu для таких потоков, Re, Pr и коэффициентом трения. Эта зависимость сравнивалась с экспериментальными результатами.